A Further Development of a Theoretical Framework for Research into Values in Mathematics Education

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Following a previous attempt to develop a theoretical framework for research into values in mathematics education, a further attempt is made using the three domains of values, culture and mathematics education. The valuing process is examined and the distinction between beliefs and values defined. The four dimensions of culture are linked with the valuing process and the elements in the teaching/learning act to construct a framework which could be the basis for further research.

In the immediate wake of the release of the results from the Third International Mathematics and Science Study (TIMSS), NSW politicians and business and community leaders deplored the fact that Australia in general, and New South Wales in particular, did not have results which placed them in the top bracket of nations taking part in the study. Their reaction and that of the students and teachers responding to the survey of beliefs included in TIMSS brought into sharp focus once again the issue of beliefs and values in mathematics education. Those quickly deciding that Australia must do better were making statements which indicated they had not considered some of the factors that might have affected the performance of Australian students such as the values placed on the subject by the culture of the students concerned or the type of test item that was used.

On a previous occasion the writer (1995) sought to develop a theoretical framework for research into values in mathematics education, on that occasion prompted by a concern to link previous research into problem solving (1992) and other research into beliefs concerning mathematics education (Southwell & Khamis, 1994). On reflecting on these results, it seemed that the critical role played by beliefs or perceptions on a person's awareness of problem solving strategies and acceptance of resolutions which conflict with those beliefs, needed further examination. Some attempt was made to identify commonly held beliefs concerning the subjects' perception of mathematics as a discipline, themselves as mathematicians and themselves as teachers of mathematics. While it does not seem that that particular research added greatly to general understanding of this aspect of mathematics teaching, it did prompt the exploration of strategies that might alleviate the anxiety and stress associated with some aspects of mathematics education.

From these general concerns, the writer was prompted to further explore the issue of beliefs and values in mathematics education and several questions were raised. What value does the educational system, community members, parents, teachers and students place on mathematics in the curriculum? What are the beliefs that underpin that value? What values are developed or clarified through study of mathematics or through teaching mathematics? What values support or derive from research into values associated with mathematics education?

The responses recorded in the TIMSS report on the beliefs of teachers and students are cause for great alarm, e.g. forty five per cent of Australian male students and forty eight percent of female students believe that "to do well in mathematics, you need lots of natural ability". Over half the Australian sample believe that "To do well in mathematics, you need lots of hard work studying at home". These responses imply that students have a narrow view of mathematics as a discipline. It appears that they see mathematics as a rigid, rule-governed set of facts and formulae that have to be memorised and applied. It also implies that these students see mathematics as a difficult subject for which a person needs to have innate talent in great degree and to have to work hard to get a good result. Such a belief might well result in students seeing mathematics as a chore which has to be endured rather than an exciting, challenging subject which will be of enormous benefit to them. These general trends are not different from previous results by Schoenfeld (1989). The responses to the items in TIMSS dealing with teaching practice indicate that the majority of classroom teachers follow a fairly "traditional" style. One

particular result of concern is the 90% response to the practice "We work from worksheets on our own". This suggests an almost total absence of group work or collaborative learning. The almost total reliance on worksheets which may be inferred from this result is most alarming.

The teacher survey focussed on two aspects of teaching. The first was teachers' preferences about teaching as a career and the other was beliefs about appreciation of their work. To the first, half the teachers surveyed said they would change their job if they were able, only one quarter of the teachers perceived any appreciation of their work from society and 64% of the teachers of the lower grade and 57% of the teachers of the higher grade felt they were appreciated by the students. It seems obvious that these beliefs are not unrelated but they do indicate a state of affairs which calls for exploration and remedy. Research into the role of values in mathematics education and the outcomes that result from it are essential fields of interest for research.

The previous framework put forward by the writer (1995) was built on the model provided by McLeod (1992) and took into consideration the definition of values given by Hill (1991). It examined the field of research into values in mathematics education from three different levels; namely, conceptual, task and output levels. These corresponded to the three domains of concepts and constructs, research methodology and classroom implications. There are some problems associated with this model and for that reason, the model has been re-examined and an alternate model suggested. This current model begins with Hill's definition and incorporates the cultural influences as well as the central features of the mathematics teaching/learning act.

Values in Mathematics Education

The writer has for some time been convinced that beliefs and values are directly related but are not the same as is implied in some of the literature. She sees the difference between the two concepts as being largely consistency and intention. A person might espouse certain beliefs but, when challenged in some way, might take action contrary to those professed beliefs. Or the other hand, a person who holds certain values, albeit based on the same beliefs, will act consistently in the face of any pressure. This is to say that beliefs are generally a subset of one's values.

Hill's definition (1991) of valuing combines three different dimensions; the cognitive, the affective and the volitional as follows:

> The three elements (of valuing) are:
> 1. A value can be described by a statement expressing a person's belief in an idea, having to do particularly with judgements of worth or obligation. Thus it has a rational or 'cognitive' element, and the belief statement involved is often referred to as a 'value judgement';
> 2. The intensity with which individuals believe certain ideas, especially those affecting the priorities they attach to certain kinds of experience, such as the moral, aesthetic and physical, indicates that there is an emotional or 'affective' element to valuing, and this is often (but not always) what people have in mind when they speak of 'attitudes';
> 3. Because one's most deeply held beliefs dispose one to act in certain ways, there is a volitional element, which leads us to refer to such beliefs as 'dispositions' or 'commitments'. We tend to make choices which are consistent with our value systems, that is, to live by the beliefs and values to which we attach highest priority. (Hill, 1991, p. 4)

These three elements can be depicted as in Figure 1. They are interconnected and interdependent.

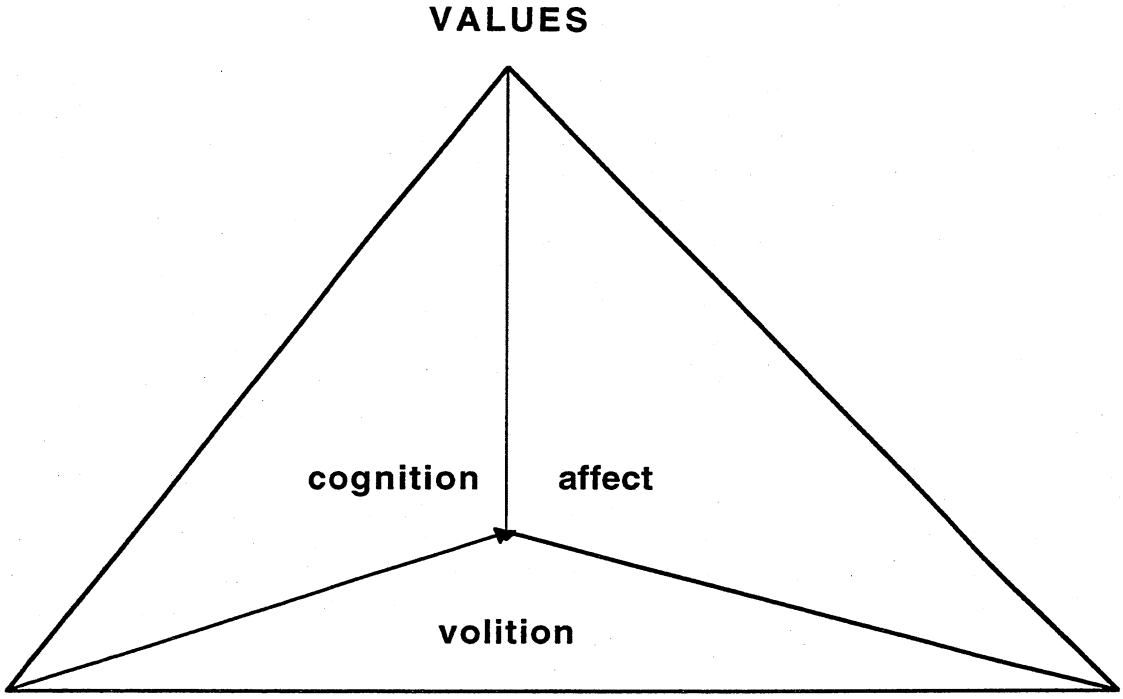

Figure 1. Three elements of Valuing

In Figure 1, values are denoted by a triangle, the three vertices of which represent the three dimensions cognition, affect and volition. They interact and unite to constitute the values a person holds and lives by.

Research in this area is regaining interest though most studies have dealt with beliefs or some other affect such as attitude (Aiken, 1970), or disposition (Brahier and Speer, 1995) rather than values.

This description of Hill's nicely ties together the relevant concepts of emotion, attitudes, beliefs, values, disposition and commitment and helps to clarify the distinctions between the closely related ideas.

Building on his description, Hill (p. 10) suggests that values education should seek to develop firstly a knowledge base which includes the traditions which have helped to mould current cultural values. Values education should also enable students to empathise with 'the perceptions and feelings of people who have been strongly committed to these traditions' and to develop skills in critical appreciation of values. Values education should also aim to encourage students 'to develop a concern for the community and the care of its members.'

What has this to do with mathematics education? What possible contribution can mathematics education make to necessary changes in values and beliefs which will enable it to contribute to national development and growth? The evidence is there in the more obvious application of values education to science and technology education, but does values education have same relevance to mathematics?

The writer maintains that attitudes towards mathematics on the part of students, teachers and parents, beliefs about mathematics and values placed on mathematics all need to change fairly radically if there is going to be a corresponding change in the value governments and the community place on the subject. The change could start from the classroom but change from that direction will be slower than if governments and the community raise the status of mathematics education in the eyes of the public. Teacher education can contribute greatly to an overall change in attitudes, beliefs and values, but if a formal examination system implies contrary values or the school executives do not keep ahead of international trends in the learning and teaching of mathematics and insist on novice teachers teaching in a routine, stereotyped way, no progress can be made.

Values are closely related to beliefs but are more complicated and encompassing. Both beliefs and attitudes can change. Beliefs are relatively stable but they can and do change with age as a person becomes more mature, has different experiences and enjoys different states. Other beliefs and attitudes which are not dependent on maturation or natural growth and development could well be more resistant to change. Some radical re-discovery or re-experience of the concept involved is usually necessary for such a change in beliefs and attitudes to take place.

An example of such a change in belief is usually necessary for primary teachers in relation to the use of calculators or computers in the classroom. The author has found, however, that a change in attitude towards calculators is harder to achieve than that towards computer usage. In one particular year, considerable resistance towards the use of calculators in the primary school among final year teacher education students was detected. They believed that students who used calculators would immediately become lazy, forget their multiplication tables and take no interest in understanding algorithms. To counter these arguments, each member of the class was asked to find a partner to work with and each pair of students was given a sheet of paper on which were written five or six key points promoting the use of calculators in schools. After reading the points, one person in each pair was asked to play the role of the teacher, while the other became an anxious parent who came to school to complain about his/her child being given a calculator in mathematics. The parent was armed with negative points and the teacher had to argue against the parent in order to justify what he or she had done. After a few minutes, the pairs were assigned new roles. The student who had been the parent became the teacher and the student who had been the teacher became a member of the school executive from whom the teacher tried to obtain permission to buy a class set of calculators. After these two role plays, all the students were asked to write a summary of what they had learnt and what they had experienced. It was interesting and encouraging that most of the students in the class had changed their beliefs about calculators and their attitude towards the use of calculators in the classroom. This was a simple activity to organise and monitor with quite astounding results.

Culture and Mathematics Education

Culture is another term which seems to be used in a number of different ways. White (1959, cited by Bishop, 1995) claims that 'the functions of culture are to relate man to his environment on the one hand, and to relate man to man, on the other' (p. 8). He goes on to categorise the components of culture under four headings:

"ideological - composed of beliefs, dependent on symbols, philosophies;
sociological - the customs, institutions, rules and patterns of interpersonal behaviour;
sentimental - attitudes, feelings concerning people, behaviour;
technological - manufacture and use of tools and implements."

White claimed that these four categories are connected and that the technological component is the one that drives the other three. The arrows from the technological domain are to denote the impact of that domain on the other three.

The social institutions of people are obviously dependent on the technology of those people. This is clear from the earliest history of man when hunting, food production and similar processes were not merely technological in nature but were also social events. Even in modern times, in many western societies, cooking, gardening, harvesting and even barn-construction are all social events. This is even more so in some less developed countries. The impact of some of the modern technologies is sometimes deplored, in that a valued social time, such as washing up in western societies, has been taken over by the dishwasher.

The ideological or philosophical component is derived from the technological component also, in that changes in technology create changes in ideology. The use of modern farm machinery gives rise to a different philosophy to that arising from the use of hand ploughs or those driven by oxen or buffaloes. Usually the latter are the plight of small subsistence farmers whereas the latest machinery indicates a more capitalist venture.

The sentimental component also is affected by the technology available, though less strongly than the other two components. Possible examples of this are the attitudes that have emerged with the development of such technologies as genetic engineering, information technology and the atomic bomb.

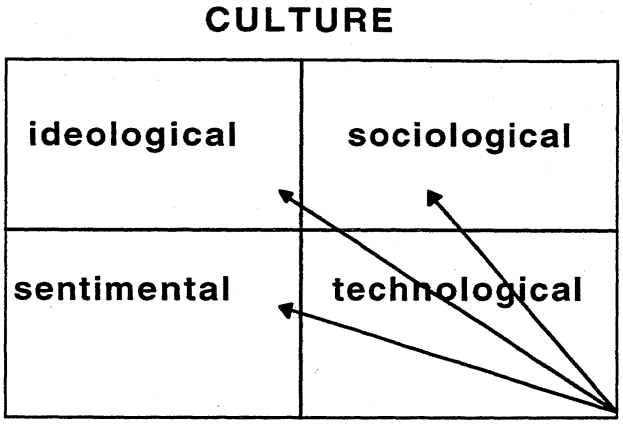

Figure 2. White's four dimensions of culture

In Figure 2, a rectangle divided into four parts is used to denote the four dimensions of culture as set out by White (1951). Although this particular representation implies a distinctiveness for the four dimensions, they are joined together by common boundaries and together they form a whole.

Delors (1996), in his report for UNESCO, introduced the notion of the four pillars of education. While he was relating these specifically to developing countries, the same statements could be made about developed countries. The aims of education must be *learning to live*, *learning to do*, *learning to be* and *learning to live together*. He places great stress - more than previously - on the last two. These two, learning to be and learning to live together, are those that enhance the humanness of the whole function of education.

It may be that governments see their national development in terms of economic growth and world trade and place little value on the maintenance of their unique culture and on the education of their citizens. Systems are designed to follow the bidding of governments, while communities and teachers are not always aware of the issues involved. Their main concern is the difficult business of living from day to day. The real importance of education for national development is not recognised and consequently education becomes a luxury rather than an essential element of a government's program. It is alarming that it appears that education is regarded more highly in developing countries than in the developed ones. A change in attitude and beliefs is required in these developed countries and this will lead in turn to greater value being given to education in national development.

Science and technology education are more highly regarded in many countries than is mathematics education. Conferences on science and technology, including values education, have been held at various times over several years. Why is it that mathematics is not so universally highly regarded?

The Mathematics Teaching/Learning Act

It is with mathematics in the classroom that this development continues. The mathematics teaching/learning act involves in turn three main elements. These are the students, the teacher and the curriculum. The students are generally taken to mean learners and teachers the instructors, though it is increasingly recognised that teachers are also learners and students are also teachers. Curriculum is used in the sense of being not only the content of the subject but also the methodology, environmental aspects and contexts. To explore the teaching/learning act, research is needed, not only on each of these elements, but also on the interactions between two or all three of these elements

since the interactions in the situation are typical of the essential characteristics of the situation. These elements and interactions can be denoted as in Figure 3.

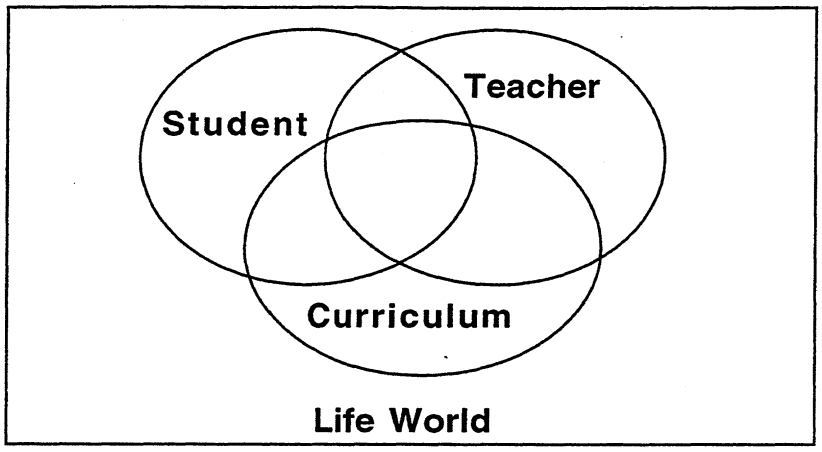

Figure 3. The Teaching/Learning Act

Figure 3 uses circles within a Venn diagram to denote the three key elements in the teaching/learning act. The three circles represent respectively the student, the teacher and the curriculum in its fullest sense. The intersection of these three sets denotes the teaching/learning act itself.

Research in this area is well founded and is approached through a variety of methodologies, including qualitative as well as quantitative methods.

Range of Research

As indicated above, very little research seems to have inquired into values in mathematics education. There is, however, a growing number of studies which investigate various aspects of beliefs and attitudes. As well as the study by Aiken (1970) mentioned previously, there have been studies more closely directed towards specific topics such as Callaghan and Watson's study (1995) on teacher attitudes towards chance and data, Taylor's (1995) on attitude to mathematics of adults returning to study using a distance mode and Kyeleve and Williams' study (1996) on the attitudes of teachers towards mathematical modelling.

Studies on beliefs related to various issues are somewhat more numerous. They include Anderson's (1996) study on beliefs concerning problem solving, Buzeika's (1996) work on teachers' beliefs and practices, Gervasoni's inquiry into students' and teachers' beliefs concerning helpful teaching practices in mathematics and Goos's (1996) study on metacognitive knowledge, beliefs and classroom mathematics. Two interesting cross-cultural studies are those by Yong (1993) who analysed differences in attitudes between performance in mathematics and science of middle grade students from African-American, Mexican-American and Chinese-America students, and Dunn (1997) whose work was on the beliefs about the mathematics instruction of preservice elementary teachers..

A variety of methodologies has been used in research into beliefs, attitudes and values. They range from the survey instrument administered to groups or individuals, such as that given by Anderson (1996) to studies similar to that of Brahier and Speer (1995) in which a number of different sets of data was collected and used. Brahier and Speer observed the children first, then had the student fill out a survey. This was followed by individual interviews and this was followed in turn by parent telephone interviews. Other methodologies used in this area are life histories (Taylor, L, 1995), reflective journals (Bobis and Cusworth , 1995) and lived experience anecdotes (Carroll, 1995).

The Outcome

When the three domains of values, culture and mathematics education are linked in some way, a structure emerges against which research into values in mathematics education could be initiated, monitored and evaluated. Such structure is shown in Figure 4.

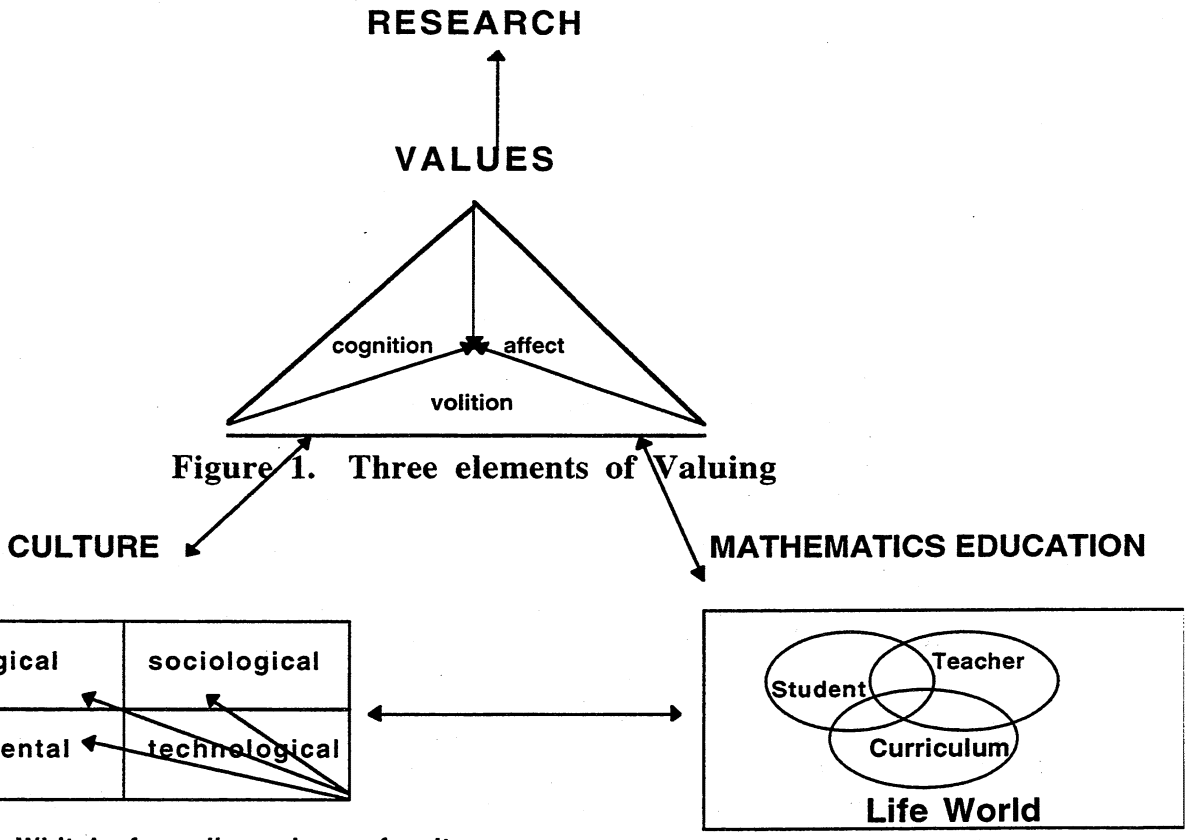

Figure 1. Three elements of Valuing

Figure 2. White's four dimensions of culture

Figure 3. The Teaching/Learning Act

Figure 4. The Accumulated Framework

The domain of values is placed at the top and central part of the diagram, and interlinked with culture, which in turn is linked with mathematics education, further linked with values. The interdependent aspect of the three domains is given by the arrows between each of the domains. The resultant of the interconnection of these three domains is research into values which makes sense of mathematics teaching in a multicultural society.

References

Aiken, L.R.Jr. (1970). Attitudes towards mathematics. *Review of Educational Research,* 40, 551-596.

Anderson, J. (1996). Some teachers beliefs and perceptions of problem solving. *Technology in Mathematics Education.* Melbourne: MERGA, pp. 30-37.

Bobis, J. & Cusworth, R. (1995). Attitudinal shifts towards mathematics of preservice teachers. In Hunting, R, Fitzsimons, Clarkson, P. and Bishop, A. (Eds.). *Regional collaboration in mathematics education.* pp109-120.

Brahier, D. and Speer, J. (1995). Mathematical disposition: An attitude check (list). In Hunting, R, Fitzsimons, Clarkson, P. and Bishop, A. (Eds.). *Regional collaboration in mathematics education.* pp. 147-156.

Buzeika, A. (1996). Teachers beliefs and practice: The chicken or the egg? *Technology in Mathematics Education,* Melbourne:MERGA, pp. 93-100.

Callaghan, R., Watson, J. Collis, K. & Moritz, J. (1996). Teacher attitudes towards chance and data. In *Galtha*, Proceedings of MERGA Conference, pp. 143 - 150. Darwin: MERGA.

Carroll, J. (1995). Primary teachers' conceptions of mathematics. In *Galtha*, Proceedings of MERGA Conference, Darwin: MERGA, pp. 151-155.

Delors, (1996). *Learning: The treasure within.* Paris UNESCO.

Dunn, T. K. (1997). Preservice elementary teachers' beliefs about mathematics instruction. Paper presented to Conference on Science, Mathematics and Technology, Hanoi, January, 1997.

Gervasoni, A. (1995). Student and teacher beliefs about helpful teaching practices in mathematics instruction. In *Galtha*, Proceedings of MERGA Conference, Darwin.: MERGA, pp. 293-299.

Goos, M. (1995). Metacognitive knowledge, beliefs and classroom mathematics. In *Galtha*, Proceedings of MERGA Conference,. Darwin: MERGA pp. 300-306.

Hill, B. V. (1991). *Values education in Australian schools.* Melbourne: ACER.

Kyeleve, J. & Williams, J. (1996). Measures of teachers' attitudes towards mathematics. In Puig, L. & Gutierrez, A. (Eds.). (1996). Proceedings of 20th Conference of IGPME. Valencia, PME, pp. 3-209-3-216.

McLeod, D.A. (1992). Research on affect in mathematics education: A reconceptualisation. In Grouws, D.A. (Ed.)(1992). *Handbook of research on mathematics teaching and learning.* New York: Macmillan, pp. 575-596.

Schoenfeld, A. H. (1989). Explorations of students' mathematical beliefs and behaviour. *Journal for Research in Mathematics Education*, 20, 238-355.

Southwell, B. (1995). Towards a theoretical framework for research in beliefs and values in mathematics education. In *Galtha*, Proceedings of MERGA Conference, Darwin.: MERGA, pp. 477 -483..

Southwell, B. and Khamis, (1994b). Beliefs about secondary Mathematics and Mathematics Education. Paper presented to AARE Conference, Nov 1994.

Taylor, J. (1995). Attitudes to mathematics of adults returning to tertiary study in the distance mode. In Hunting, R, Fitzsimons, Clarkson, P. and Bishop, A. (Eds.). *Regional collaboration in mathematics education.* pp.685-694.

Taylor, L. (1995). Documenting and exploring ethnographic mathematical life histories. In Hunting, R, Fitzsimons, Clarkson, P. and Bishop, A. (Eds.). Conference proceedings on *Regional Collaboration in Mathematics Education.* pp. 695-701.

White, L. (1959). *The evolution of culture.* London: McGraw-Hill, (p. 8) cited in Bishop, A. (1988). *Mathematics enculturation.* Dordrecht: Kluwer, p. 16.

Yong, F.L. (1993). Attitudes towards Mathematics and Science of African-American, Mexican -American and Chinese- American middle grade Students identified as gifted. *Focus on learning problems in mathematics*, 15, 1, 52-61.